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MAGNETIC RIPPLE CORRECTION IN TANDEM MIRRORS BY FERROMAGNETIC INSERTS*

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ABSTRACT

Magnetic ripple of 1% or more caused by discrete solenoid coils in the central cells of tandem mirrors may severely affect the MHD stability. The ripple amplitude can be reduced by an order of magnitude by ferromagnetic annuli inserted within the coils at the regions of ripple maxima. The inserts need not affect the accessibility, coil diameter, or capital cost, since large quantities of steel are required within the coils for the neutron blanket and shield. Design of the ripple correction is simplified and linearized by the cylindrical geometry and by the saturation of the ferromagnetic steel.

INTRODUCTION AND SUMMARY

Magnetic ripple caused by discrete solenoid coils in the central cells of tandem mirrors may severely restrict the plasma beta for MHD stability if the ripple amplitude is much larger than 1%. The ripple problem tends to be more severe as the spacing between solenoid coils is increased for access and as the coil diameter is reduced for economy. Solenoid design must include some type of economical ripple correction compatible with the various requirements of the coils and also of the neutron blanket within the coils.

It was found that at little or no extra expense, the ripple amplitude could be reduced by an order of magnitude through the insertion of ferro-magnetic annuli at the locations of the ripple maxima. In computer testing it was discovered that the induced magnetization of the iron reduced the field at the ripple maxima and increased the field at the ripple minima. In one example, the ripple amplitude was reduced from 9.5% to 0.5% by iron inserts within the solenoid coils. Ripple correction was designed for several coil geometries, which the paper will describe.

Because in most fusion reactor designs large quantities of steel are required within the magnet coils for the neutron blanket, reflector, and shield, the iron needed for ripple correction was already available. Thus, to correct the ripple only necessitated that certain sections of steel blanket, reflector, and shield be specified ferromagnetic, and other sections be specified non-magnetic.

Ripple correction by ferromagnetic inserts has previously been designed for tokamak reactors.[1] It appears that the ripple correction is easier to achieve in the central cell of a tandem mirror, compared to a tokamak, because of the linear geometry. Also, the computational design was greatly simplified for the tandem mirror because of the spatially constant induced magnetization. This constant magnetization was due to the cylindrical geometry and the saturation of the iron. Optimization of coil currents and the iron content of the inserts was tractable by linear matrix algebra.

The effect of nonuniform blanket temperature on the ripple has been analyzed and found to be very small.

EFFECT OF RIPPLE ON MHD STABILITY

Computations[2] in 1983 using the TEBASCO code[3] indicated that the plasma beta allowable for MHD stability in the central cells of tandem mirrors can be severely restricted by magnetic ripple due to discrete solenoid coils. In a high-beta plasma this effect is magnified because the ripple percentage in the plasma magnetic field is larger than the ripple percentage in the vacuum magnetic field. In reactor relevant examples, it was found that the ripple percentage of the vacuum field should be not more than 1% if this penalty cannot be accepted.

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This conclusion ultimately may be modified because of more recent computations, not yet published, which include the stabilizing effect of the walls. However, the 1983 results motivated a series of solenoid designs which minimized the magnetic ripple while being compatible with engineering requirements of the superconducting coils and of the neutron blanket and shield.

TYPES OF SOLENOID COILS

Two types of solenoid coils will be considered in this paper, which I will refer to as discrete coils and sheet coils, respectively. In early versions of tandem mirror reactors, such as the Mirror Advanced Reactor Study (MARS),[4] the solenoid of the central cell was designed as a series of separate coils at intervals of about three meters. Fig. 1 shows an example of such a solenoid, composed of 22 discrete coils. The three-meter spacing was thought to be required to permit access to the neutron blanket and shield for maintenance, replacement, and heat transfer. Consequently, the magnetic ripple percentage $(B_{max} - B_{min})/B_{max}$ was in the range 4% to 10%, depending upon conditions.

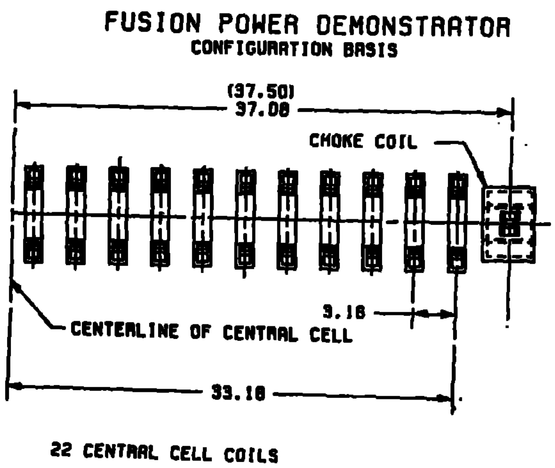


Fig. 1. Coil layout of FPD solenoid A19, with discrete coils at intervals of 3.16 meters.

Using discrete coils, the ripple percentage can be reduced either by increasing the coil diameter at increased capital cost or by reducing the spacing between coils at the sacrifice of accessibility. Both of these options are undesirable for the reasons mentioned. A third possibility, to be explained in this paper, is to correct the ripple with ferromagnetic annuli inserted within the coils.

Recently, under a more aggressive design philosophy, sheet coils have been designed in which the conductor bundle consists of a continuous sheet in a module perhaps ten meters long. Such a solenoid, shown by Fig. 2, is lighter in weight because of the smaller outside diameter and higher current density. It also has the advantage of having no magnetic ripple except at the gaps between modules necessary for access. This design is now being studied for accessibility and other engineering requirements.

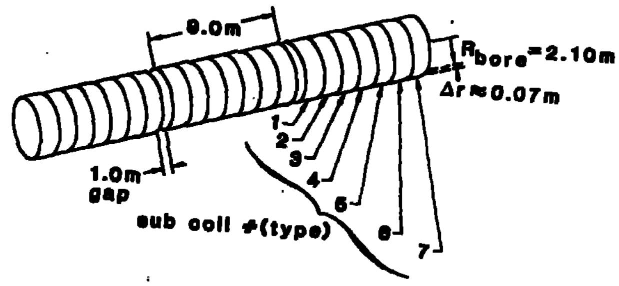


Fig. 2. Coil layout of FPD solenoid A21, with sheet coils in an assembly 9 meters long with a 1-meter gap between assemblies. Each assembly consists of 7 sub-assemblies.

MATHEMATICAL MODEL FOR RIPPLE CORRECTION

Normally the computation of magnetic fields involving ferrromagnetic materials require a non-linear code such as *GFUN-3D*.[5] However, because of simplifications applying to the central cells of tandem mirror reactors a mathematical model has been devised without additional approximations that can be executed by a linear code such as EFFI,[6] which is much easier to set up and to operate than the non-linear codes. The mathematical simplifications are the following:

1. Cylindrical symmetry.

2. Saturated iron. Therefore, the magnetization vector $\underline{M}$ is constant in magnitude and independent of the applied field.

3. $B_z >> B_r$. Therefore, the radial component of $\underline{M}$ can be neglected. $\underline{M}$ is essentially constant in direction as well as magnitude.

4. Magnetization currents. Since $\underline{M}$ is constant, there are no magnetization currents within the iron. At the surface of the iron, where $\underline{n}$ is the unit vector normal to the surface, there is a surface magnetization current of the following density per unit length:

$$\underline{J} = (-\underline{n} \times \underline{M})/\mu_0 \qquad \text{A/m} \qquad (1)$$

Since $\underline{M}$ is always axial and because of cylindrical symmetry, $\underline{J}$ is azimuthal. The iron insert will be some kind of annulus with surface magnetization currents clockwise on the inside surface and counter-clockwise on the outside surface. There will be no magnetization currents on the end surfaces where $\underline{n} \times \underline{M} = 0$.

5. Biot-Savart's Law and EFFI. Since we know the magnetization currents, we can replace the iron insert with two concentric coils equivalent to the surface magnetization currents. The magnetic field $\underline{B}(r,z)$ due to these currents can be computed at any point in space by Biot-Savart's Law or by the program EFFI. On the axis of symmetry, Biot-Savart's Law can be integrated to provide an analytic solution for $\underline{B}(0,z)$.

ANALYTIC SOLUTION

Text books[7] show that the magnetic field on the axis of a cylindrical solenoid of length L is obtained by integration of Biot-Savart's Law in the range $-L/2 < z < +L/2$:

$$B(0,z) = \mu_0 J/2 (\cos \phi_1 - \cos \phi_2) \qquad (2)$$

Here the angles ϕ_1 and ϕ_2 are defined to be subtended at the point $(0,z)$ by the two ends of the cylindrical coil, i.e.,

$$\tan \phi_1 = r_c/dz_1,$$

where $dz_1 = L/2 - z;\ dz_2 = -L/2 - z.$ (3)

In the above, r_c is the coil radius (i.e., the radius of the inner or outer surface of the iron annulus) and ϕ_2 is defined similarly using dz_2. We use the trigonometric identity cos = 1/sec = 1/sqrt(1+tan**2).

RIPPLE CORRECTION OF A DISCRETE COIL SOLENOID

As a first example, I used an early version of the Fusion Power Demonstrator (FPD) design,[8] illustrated by Fig. 1, in which the central cell solenoid consisted of 22 discrete coils at intervals of 3.16 meters. EFFI computations showed that the vacuum magnetic field ripple was 9.5%, which was considered not acceptable for high-beta MHD stability.

I assumed that the neutron blanket and shield of the FPD would be essentially the same as the design by the University of Wisconsin for MARS:

Blanket materials:
- LiPb neutron absorber
- HT-9 steel structure

Blanket radii:
- $59 < r < 98$ cm

Reflector and shield materials:
- $100 < r < 154$ cm: Fe-1422 steel water-cooled
- $154 < r < 184$ cm: Pb, etc.

Ripple correction is most effective if the ferromagnets are as close as possible to the plasma, i.e., just outside the blanket, which is mostly LiPb. The reflector and shield in the region $100 < r < 154$ cm were ideal for this purpose, since they consisted of 95% steel. For each coil module 3.16 meters long, I specified a ferromagnetic section 1.75 meters long centered within each coil and a non-magnetic section 1.41 meters long between coils.

I assumed that the exact value of the saturated magnetization M was adjustable by design within limits, since the ferromagnets need not be 100% iron. Using EFFI,[6] I adjusted M to minimize the residual ripple. Fig. 3 shows that the ripple on axis was reduced from 9.5% to 0.45% when I set M = 1.10 T.

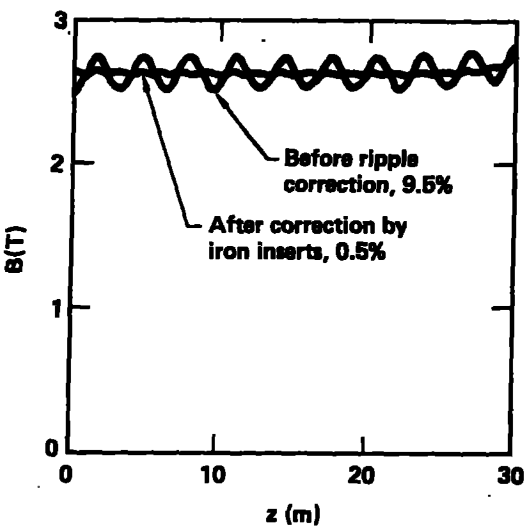


Fig. 3. Graphical EFFI output showing residual ripple, coil set A19.

Table I shows that the ripple off axis was also very small within the volume occupied by the central cell plasma. This is because the plasma radius is small in comparison with the radii of the coils and the ferromagnets.

The residual ripple could be further reduced by adjusting the length L of the ferromagnets or by shaping their surfaces. However, the above results were satisfactory for the purpose of FPD.

TABLE I
Residual magnetic ripple
as a function of radius
within the plasma of radius 45 cm

R (cm)	% Ripple	Br/Bz, maximum
0	0.45%	0.
15	0.51%	0.875 E-3
30	0.72%	1.61 E-3
45	1.16%	3.094 E-3

The required magnetization M = 1.10 T is consistent with a combination of 95% HT-9 steel (M > 1.15 T) and 5% cooling water. These are the fractions specified for steel and water in the MARS design.[4] The nonmagnetic sections of the reflector and shield could be Fe-1422, also as specified in the MARS design.

RIPPLE CORRECTION FOR A SHEET-COIL SOLENOID

Each module of the sheet-coil solenoid shown by Fig. 2 consists of seven sub-modules numbered 1 through 7. The current in each sub-module was adjusted to minimize the ripple, which was 4.4% in version A21, with one-meter gaps between modules for access. In this version the maximum magnetic field, shown by Fig. 4(a), was within sub-modules 1 and 7 because of the extra current used to strengthen the field in the gaps.

I installed iron inserts 2 meters long at the locations of maximum magnetic field, adjacent to the gap. I then readjusted the currents in each of the seven submodules and readjusted the magnetization M to reduce the ripple to 0.8% as shown in Fig. 4(b).

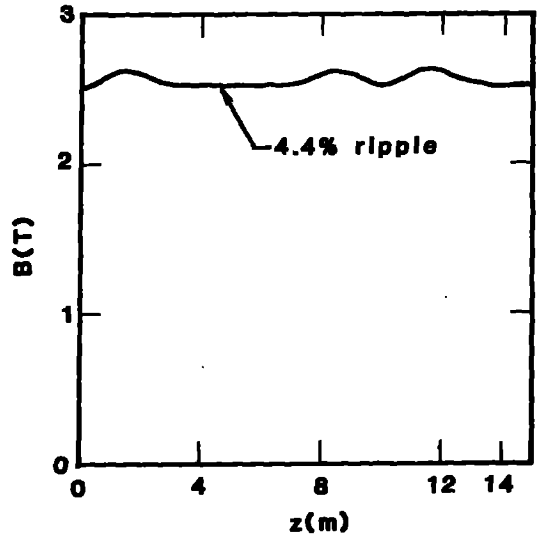
RIPPLE CORRECTION BY MATRIX ALGEBRA

Optimization of coil currents and magnetic iron for ripple correction can be greatly facilitated by linear matrix algebra. The magnetic field $\underline{B}(r,z)$ is a linear function of coil currents and magnetization currents and is a nonlinear function of the dimensions of the coils and of the magnetic iron inserts. Ripple correction requires optimization of about 10 independent variables, which can be laborious by trial-and-error.

However, trial-and-error is not necessary to optimize the independent variables of which $\underline{B}(r,z)$ is a linear function. This part of the optimization can be done precisely by matrix algebra. Therefore, the optimization is most easily done by using iterative guess-work to choose the dimensions and by using matrix inversion to determine the currents after each choice of dimensions.

EXAMPLE

As an example, I will use Bulmer's FPD sheet-coil assembly A21 shown by Fig. 3, which consists of six solenoids 9 meters long with gaps of 1 meter between solenoids. To reduce the ripple to 4.4%, each solenoid consists of 7 sections each of which is adjustable in current density.



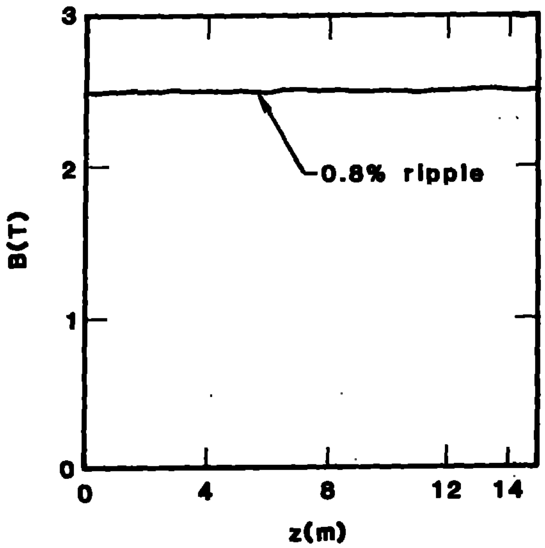


Fig. 4. Graphical EFFI output showing residual ripple, coil sets A21 and A21G.
(a) No iron. 4.4% ripple.
(b) With iron inserts and with re-optimized coil currents. 0.8% ripple.

In version A21G I add iron inserts adjacent to the gap and readjust the seven coil currents and four surface magnetization currents to reduce the residual ripple to 0.8%. By symmetry the seven coil currents are specified by four unknown currents. The four magnetization currents are equal in density but in opposite directions, and are specified by one additional unknown.

MATRIX OPERATIONS

The sheet coil conductors are only 7-cm thick and can be replaced with a surface current of zero thickness. For coil current J(i) Amperes/linear meter, the magnetic field on the axis of symmetry at point j is

$$B(j) = a(i,j)\ J(i) \tag{4}$$

where the coefficient a(i,j) is known analytically by equations (2) and (3).

We have a total of at least 66 unknown currents in the FPD solenoid, which consists of 6 or more assemblies, each involving 7 coil currents and 4 magnetization currents. By symmetry, these can be combined into four unknown coil currents and one unknown magnetization current. Equation (4) can be considered to be the sum of five terms, where the summation is indicated by the repeated index i.

If we specify $\underline{B}(j)$ = 2.5 Tesla at five different locations z(j), equation (4) becomes a system of five simultaneous equations with the five unknowns $\underline{J}(i)$. In matrix notation, the five equations are:

$$\underline{B} = A\ \underline{J} \tag{5}$$

where $\underline{B}$ and $\underline{J}$ are 5-dimensional vectors and A is a 5 x 5 matrix with matrix elements a(i,j). The solution is

$$\underline{J} = A^{-1}\ \underline{B} \tag{6}$$

where A^{-1} is the inverse of matrix A.

PROGRAM MAT4

To carry out this computation, I wrote a Fortran program MAT4 which carries out the following operations:

1. Subroutine Cosine computes the matrix elements a(i,j) by equations (2) and (3) for each of the 66 currents in the FPD solenoid at each of the five points z(j). These contributions by coils with equal currents are added appropriately by MAT4 to compile the 5 x 5 matrix A.

2. The IMSL Subroutine LGINF inverts the matrix A to produce A^{-1}. Then MAT4 computes the five-dimensional $\underline{J}$ vector by the matrix multiplication of equation (6).

3. The linear current densities J(i) are converted to area current densities j(i) by dividing by the coil thickness (7 cm).

4. Finally, to verify the correctness of the matrix inversion, MAT4 verifies by matrix multiplication that

$$A\ A^{-1} = I, \text{ the unit matrix} \tag{7}$$

The output j(i) of MAT4 is fed into EFFI to compute the magnetic field $\underline{B}$(r,z). The computer work is guaranteed to be correct if the EFFI output shows $\underline{B}$(j) = 2.50 Tesla at the five specified points.

RESULTS

Fig. 4(a) shows the graphical EFFI output showing the 4.4% ripple of coil set A21, which had been manually optimized without iron. For comparison Fig. 4(b) shows the 0.8% residual ripple of A21G, corrected by iron inserts and re-optimized by matrix algebra. The re-optimization is indicated by Table 1.

TABLE II
Optimized Coil Currents and Dimensions

	A21 no iron	A21G with iron
PERCENT RIPPLE	4.4%	0.8%
j (A/m2)	4.697E7	4.503E7
	2.145E7	2.481E7
	3.053E7	2.561E7
	2.693E7	3.052E7
COIL DIMENSIONS		
radius (m)	2.1	2.1
length of assembly (m)	9.	9.
thickness (m)	.07	.07
gap between assemblies	1.	1.
number of assemblies	6.	6.
coils each assembly	7.	7.
length of each coil	9/7	9/7
IRON INSERTS in A21G		
position	adjacent to gaps, each end	
length (m)	2.	
inside radius (m)	1. (MARS design)	
outside radius (m)	1.54 " "	
surface magnetization currents $\underline{J}$ (A/m)	5.393E5	
average magnetization (Tesla) = $\mu_0\ \underline{J}$	0.68	
saturation strength of HT-9 (Tesla)	1.15	
required percentage of HT-9 in insert	59%	

EFFECT OF NON-UNIFORM TEMPERATURE OF FERROMAGNETIC INSERTS

In some reactor designs the temperature of the neutron reflector and shield is not uniform. Therefore, the magnetization M of the saturated iron is non-uniform and there will be magnetization currents within the iron as well as on the surfaces.

Using some of the techniques of Attaya,[9] I have modeled this situation, including magnetization currents within the iron, assuming temperatures in the range from 330-500°C, as in the MARS design. As shown in Table III, the temperature differences have a very small effect on the magnetic ripple under these conditions:

TABLE III
Effect of azimuthal temperature gradients upon magnetic ripple correction

y-coordinate	Residual Ripple
+ 0.3 m (near the cooler iron)	1.10%
0. (on axis)	0.79%
- 0.3 m (near the warmer iron)	0.89%

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